Answer Key to Integral Calculus - 1

Level – 0	Based on CBSE Pattern

1. $x-\log\left|(2x+3)^2\right|+C$ **2.** $\log\left|\left(x^2+3x\right)\right|+C$ **3.** $\frac{x^2}{2}-x+3\log|x+1|+C$

4. $\frac{x^3}{3}+C$ **5.** $\log\left|(x+\sin x)\right|+C$ **6.** $\tan\frac{x}{2}+C$

7. $\frac{\tan^5 x}{5}+\frac{\tan^3 x}{3}+C$ **8.** $x+C$ **9.** $-2\cos\frac{x}{2}+2\sin\frac{x}{2}+C$

10. $2\left[\frac{x\sqrt{x}}{3}-\frac{x}{2}+\sqrt{x}-\log\left|\left(\sqrt{x}+1\right)\right|\right]+C$ **11.** $a\sin^{-1}\left(\frac{x}{a}\right)-\sqrt{a^2-x^2}+C$

12. $\frac{4}{3}x^{3/4}-\log|\left(1+x^{3/4}\right)+C$ **13.** $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}+C$

14. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$ **15.** $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4t-3}{3}\right)+C$

16. $3\sqrt{x^2+9}-\log\left|x+\sqrt{x^2+9}\right|+C$

17. $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|x-1+\sqrt{5-2x+x^2}\right|+C$ **18.** $\frac{1}{4}\left[\left|\log\left|x^2-1\right|-\log\left|x^2+1\right|\right|\right]+C$

19. $\frac{1}{4}\log\left|\frac{1+x}{1-x}\right|-\frac{1}{2}\tan^{-1}x+C$ **20.** $\frac{x-a}{2}\sqrt{2ax-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+C$

21. $\sin^{-1}x.\frac{x}{\sqrt{1-x^2}}+\log\left|\sqrt{1-x^2}\right|+C$ **22.** $-\frac{1}{2}\sin 2x-\sin x+C$

23. $\tan x-\cot x-3x+C$ **24.** $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$

25. $2\sin x+x+C$ **26.** $\frac{1}{2}\sec^{-1}\left(x^2\right)+C$

27. $\frac{1}{7}\log\left|\frac{x-2}{x+2}\right|+\frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}}+C$ **28.** $\frac{1}{a^2-b^2}\left[a\tan^{-1}\frac{x}{a}-b\tan^{-1}\frac{x}{b}\right]$

29. $\log\left|\frac{\sqrt{x-3}}{(x-1)^{1/6}(x+2)^{1/3}}\right|+C$ **30.** $xe^{\tan^{-1}x}+C$

31. $a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}}+\tan^{-1}\sqrt{\frac{x}{a}}\right]+C$ **32.** $\frac{e^{-3x}}{24}\left[\sin 3x-\cos 3x\right]+\frac{3e^{-3x}}{40}\left[\sin x-3\cos x\right]+C$

33. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+C$ **34.** A **35.** C **36.** A

37. D **38.** C **39.** D **40.** D **41.** D **42.** A

Level – 1	Daily Tutorial Sheet 1 to 6

1. (i) $\frac{x^2}{2}+\ell nx-2x+C$ (ii) $\frac{x^{7/2}}{7/2}+3.\frac{x^{5/2}}{5/2}+\frac{4x^{1/2}}{1/2}+C$

(iii) $\frac{1}{(b-c)}\left(\frac{2}{3}\frac{(ax+b)^{3/2}}{a}+\frac{2}{3}\frac{(ax+c)^{3/2}}{a}\right)+C$ (iv) $\frac{x^3}{3}+\frac{4x^2}{2}+6x+7\log|x-1|+C$

(v) $\frac{1}{4}\left[\frac{1}{3}\sin 3x+3\sin x\right]+c$ (vi) $\tan x-\cot x+C$

2. (i) $\sqrt{2}\times\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]+C$ (ii) $\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+C$

(iii) $\frac{1}{4}\left(x+\frac{\sin 6x}{6}+\frac{\sin 4x}{4}+\frac{\sin 2x}{2}\right)+C$ (iv) $-\frac{4}{3}(1-x)^{3/2}+\frac{2}{5}(1-x)^{5/2}+C$

3. (i) $\frac{(2x+1)^{3/2}}{3}-4\frac{(2x+1)^{1/2}}{1}+C$ (ii) $\begin{cases} x+C \ \forall\ x\in\left(2n\pi-\frac{\pi}{4},\ 2n\pi+\frac{3\pi}{4}\right) \\ -x+C \ \forall\ x\in\left(2n\pi+\frac{3\pi}{4},\ 2n\pi+\frac{7\pi}{4}\right) \end{cases}$

(iii) $\frac{1}{4}\left(x-\frac{\sin 4x}{4}-\frac{\cos 4x}{4}\right)+C$ (iv) $\frac{\tan\frac{x}{2}}{1/2}-x+C$

(v) $\tan x+\sec x+C$ (vi) $-\cot x-\operatorname{cosec} x+C$

4. (i) $\tan^{-1}e^x+C$ (ii) $\frac{1}{3}\ell n\,|1-x^3|+\frac{1}{3(1-x^3)}+C$

(iii) $\frac{1}{4}\ell n\left|x^4+\sqrt{1+x^8}\right|+C$ (iv) $\frac{\left(\tan^{-1}x^2\right)^2}{2}+C$

(v) $\frac{-1}{2(\sin x-\cos x)^2}+C$ (vi) $\frac{1}{2a}(1-p)^{-1}\left(ax^2+b\right)^{1-p}+C$

5. (i) $2\sqrt{\sin x}-\frac{2}{5}(\sin x)^{5/2}+c$ (ii) $=\sec x+\cos x+c$

(iii) $\frac{\sec^6 x}{6}+c$ (iv) $\frac{\sec^7 x}{7}-\frac{\sec^5 x}{5}+c$ (v) $\tan(\log x)+c$

(vi) $\sqrt{x^2-a^2}-\frac{a^2}{\sqrt{x^2-a^2}}+c$

6. $\frac{1}{2}\ln|e^{2x}+1|+c$ **7.** $\frac{1}{\sin(b-a)}\ln\frac{\sin(x-b)}{\sin(x-a)}+c$

8. $\ln|\sec^2 x+\sec x\tan x|+C$ **9.** $\frac{1}{\sin(b-a)}\ln\left|\frac{\sec(x-a)}{\sec(x-b)}\right|+C$

10. $\frac{2}{3(a-b)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+C$ **11.** $2\left[\frac{(x+a)^{5/2}}{5}-a\frac{(x+a)^{3/2}}{3}\right]-2\left[\frac{(x+b)^{5/2}}{5}-b\frac{(x+b)^{3/2}}{3}\right]+C$

12. $-2\cot 2x + C$ **13.** $x\cos a + \sin a \ln|\sin(x-a)| + C$

14	15	16	17	18	19	20	21	22.	23.
B	D	B	C	A	B	C	A	B	A
24	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
D	B	C	CD	D	B	CD	A	C	C
34	**35**	**36**	**37**	**38**	**39**	**40**		**41**	
C	A	C	C	B	D	AB		B	

42. (i) $\frac{1}{2}\log|x^2+x+1| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$

(ii) $(x^2+4x+1) - 12\left[\frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|\right] + C$

(iii) $\frac{1}{2}\left[\frac{2(x^2+x+1)^{3/2}}{3} + \frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|\right] + C$

(iv) $\frac{1}{2}x + \frac{1}{4}\log(2x^2+4x+1) + \log\left|(x+1)+\sqrt{x^2+2x+\frac{1}{2}}\right| + C$ (v) $\frac{1}{2}\tan^{-1}\left(\tan(x)+\frac{1}{2}\right) + C$

43. (i) $\frac{1}{3}\log\left|\frac{3+\tan\left(\frac{x}{2}\right)}{3-\tan\left(\frac{x}{2}\right)}\right| + C$ (ii) $\frac{12}{13}x - \frac{5}{13}(3\cos x + 2\sin x) + C$

(iii) $\frac{6}{5}x + \frac{3}{5}\log|\sin x + 2\cos x + 3| - \frac{8}{5}\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{2}\right) + C$ (iv) $-\frac{1}{2(2\tan x+3)} + C$

(v) $\ln\left|\tan x + 2\right| + C$

44. (i) $\frac{1}{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + C\right]$

(ii) $\frac{1}{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + C\right]$

(iii) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\log\left(\frac{\tan x + 1 - \sqrt{2\tan x}}{\tan x + 1 + \sqrt{2\tan x}}\right) + C$

(iv) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot x - 1}{\sqrt{2\cot x}}\right) - \frac{1}{2\sqrt{2}}\log\left(\frac{\cot x + 1 - \sqrt{2\cot x}}{\cot x + 1 + \sqrt{2\cot x}}\right) + C$ (v) $-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\tan x - 1}{\sqrt{2}+\tan x + 1}\right| + C$

45. (i) $\frac{x^2}{2}\log(x+1)-\frac{1}{2}\log(x+1)+\frac{x}{2}-\frac{x^2}{4}+c$ (ii) $-\frac{\log x}{x}-\frac{1}{x}+c$

(iii) $x\tan x-\ell n\,|\sec x|-(x\sec x-\ell n\,|\sec x+\tan x|)+c$

(iv) $\frac{x}{2}\left(\cos(\log x)+\sin(\log x)\right)+c$ (v) $x\log\left(x+\sqrt{x^2+a^2}-\sqrt{x^2+a^2}\right)+C$ (vi) $\frac{x}{1+\log x}+c$

46. (i) $\frac{e^x}{1+x^2}+c$ (ii) $-e^x\cot\frac{x}{2}+c$ (iii) $\frac{2}{3}x^{3/2}(\log x)^2-\frac{8}{9}x^{3/2}\log x+\frac{16}{27}x^{3/2}+c$

(iv) $\frac{a^x x^2}{\ln a}-\frac{2xa^x}{\ln a}+\frac{2a^x}{(\ln a)^2}+C$

(v) $x\log\left(\sqrt{1-x}+\sqrt{1+x}\right)+\frac{x}{2}-\frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}\frac{x}{2}\right]+C$ (vi) $\sin x\log\left(\tan\frac{x}{2}\right)-x+c$

47. $2\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+C$ **48.** $\frac{1}{2}\tan^{-1}(2\tan x)+C$ **49.** $\frac{1}{5}\ln|\tan\theta-2|-\frac{2}{5}\ln|2\tan\theta+1|+C$

50. $\frac{1}{p^2+q^2}\left[q\log|p+q\tan x|-\frac{q}{2}\log|\sec^2 x|\right]+C$ **51.** $\frac{1}{2}lm\left|\frac{\tan x}{\tan x+2}\right|+C$

52. $\frac{1}{2}\ln(2\cos x+3)+\frac{4}{\sqrt{5}}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)+C$ **53.** $\frac{x}{2}+\frac{1}{2}\ell n\,|\sin x-\cos x|+c$

54. $a\,sin^{-1}\frac{x}{a}-\sqrt{a^2-x^2}+c.$ **55.** $\frac{a^2}{2}\sin^{-1}\left(\frac{x^2}{a^2}\right)+\frac{1}{2}\sqrt{a^4-x^4}+c$

56. $\frac{\ln(1-x^2)}{2}+\frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}+C$ **57.** $x\log|1+x^2|-2x+2\tan^{-1}x+C$

58. $-\cos x\log(\sec x+\tan x)+x+C$

59. $x\cos a-\sin a\ln|\sec(x-a)|+C$ **60.** $\frac{x\cos^{-1}x}{2}+\frac{\sqrt{x^2-1}}{2}+C$

61	62	63	64	65	66	67	68	69	70
A	A	B	C	B	D	B	B	C	D
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
C	B	AD	C	A	D	D	C	B	A
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
C	A	A	B	C	A	A	A	D	B

92. $I_n = \frac{e^{ax}}{a^2+1}\sin^{n-1}x\left[a\sin x - \cos x\right] - \frac{1}{a^2+1}\left[an\, I_n - e^{ax}\sin^{n-1}x\right]$

93. $t + \frac{a}{\sqrt{2}}\ln\left|\frac{t-\sqrt{2}a}{t+\sqrt{2}a}\right| + C$ where $t = \sqrt{a^2 - x^2}$

94. $x\left[\log\log x - \frac{1}{\log x}\right] + c$ **95.** $-\frac{1}{3a^2}\left(\frac{a^2}{x^2} - 1\right)^{3/2} + C$ **96.** $-\frac{2}{\sin a}\sqrt{\cos a + \cot x \sin a} + C$

97. $\log\left|\cos x + \sqrt{\cos^2 x - \frac{1}{2}}\right| + \frac{1}{2}\log\left|\frac{\sqrt{\cos 2x} - 1}{\sqrt{\cos 2x} + 1}\right| + C$

98. $\frac{2}{(a^2-b^2)^{3/2}}\left[a\theta - \frac{b\tan\theta}{1+\tan^2\theta}\right] + c$ where $\tan^2\theta = \frac{(a-b)}{(a+b)}\tan^2\frac{x}{2}$

99. $\frac{e^x(x+1)}{x^2+1} + c$ **100.** $x^2\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{2a-x}{a}}\right) - 2\sqrt{a}\left[\frac{(2a+x)^{5/2}}{5} + 4a^2\sqrt{2a+x} + \frac{4a}{3}(2a+x)^{3/2}\right] + C$

101. $-3t - \frac{3}{4}\ln\left|\frac{t-1}{t+1}\right| + \frac{3}{2}\tan^{-1}t + C$ where $t = \left(\frac{x+1}{x+2}\right)^{1/4}$

102. $-\cos\alpha\sin^{-1}\left(\frac{\cos x}{\cos\alpha}\right) - \sin\alpha\ln\left|\sin x + \sqrt{\sin^2 x - \sin^2\alpha}\right| + C$

103.	**104.**	**105.**	**106.**	**107.**	**108.**	**109.**	**110.**	**111.**	**112.**
C	C	C	D	B	B	A	C	A	B
113.	**114.**	**115.**	**116.**	**117.**	**118.**	**119.**	**120.**	**121.**	**122.**
B	B	C	D	A	C	A	D	D	B
123.	**124.**	**125.**	**126.**	**127.**	**128.**	**129.**	**130.**	**131.**	**132.**
B	B	C	D	D	B	A	B	B	C
133.	**134.**	**135.**	**136.**	**137.**	**138.**	**139.**		**140.**	
A	B	B	B	A	D	B		A	
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**
0	**1.25**	**8**	**1.25**	**2**	**60**	**4**	**21**	**22**	**1**
151	152	153	154	155					
5	**40.3**	**5**	**12**	**2**					

JEE Main (Archive)	Daily Tutorial Sheet – 1 to 2

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
(D)	(B)	(C)	(C)	(A)	(A)	(D)	(A)	(A)	(D)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
AC	C	C	D	B	D	B	A	B	B
21	**22**	**23**	**24**	**25**	**26**				
D	D	D	C	D	A				

JEE Advanced (Archive)	Daily Tutorial Sheet – 1 to 2

1.	2.	3.	4.
C	C	A	C

5. $A = -3/2, B = \frac{35}{36}, C \in R$

6. $\frac{1}{6(m+1)}.(2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m} + c$

7. $2\left[\cos^{-1} x - \log|1+\sqrt{1-x}| - \frac{1}{2}\log|x|\right] + c$

8. $-2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x(1-x)} + c$

9. (i) $4\sin\frac{x}{4} - 4\cos\frac{x}{4} + c$ (ii) $-2\left\{\sqrt{1-x} - \frac{2}{3}(1-x)^{3/2} + \frac{1}{5}(1-x)^{5/2}\right\} + c$

10. $\frac{1}{b^3}\left(a+bx - 2a\log(a+bx) - \frac{a^2}{a+bx} + c\right)$

11. $-\frac{\cos 4x}{16} - \frac{\cos 2x}{8} + \frac{\cos 6x}{24} + \sin 2x + \tan x - 2x + \frac{3x}{128} - \frac{\sin 4x}{128} + \frac{\sin 8x}{1024} + C$

12. $\frac{1}{2}\tan^{-1}(x^2) + c$

13. $\frac{1}{2}\log(\sin x - \cos x) + \frac{1}{2}x + c$

14. $\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + \frac{4}{3}x^{1/2} - \frac{12}{5}x^{5/12} + \frac{1}{2}x^{1/3} - 4x^{1/4} - 7x^{1/6} - 12x^{1/12}$
$+(2x^{1/2} - 3x^{1/3} + 6x^{1/6} + 11)\ln(1+x^{1/6}) + 12\ln(1+x^{1/2}) - 3[\ln(1+x^{1/6})]^2 + c$

15. $\sqrt{2}\tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + c$

16. $-\log|\cot x + \sqrt{\cot^2 x - 1}| + \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+\sqrt{1-\tan^2 x}}{\sqrt{2}-\sqrt{1-\tan^2 x}}\right| + c$

17.(1)

18. $(x+1)\tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{4}\log(4x^2+8x+13) + c$

19. $\frac{1}{2}\sin 2\theta \ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) + \frac{1}{2}\ln(\cos 2\theta) + c$

20. $\frac{2}{\pi}\left[\sqrt{x-x^2} - (1-2x)\sin^{-1}\sqrt{x}\right] - x + c$

21. $\frac{e^x}{(x+1)^2} + c$

22. $x\sin x + \cos x - \frac{\cos 2x}{4} + c$

23.(C)

24. $I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+1| + \frac{3}{2}\tan^{-1}x + \frac{x}{x^2+1} + C$

25. $\log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + c$